PROMOTING CREAVITY IN LEARNING MATHEMATICSUSING OPEN-ENDED PROBLEMS

By Tatag Siswono

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Tatag Yuli Eko Siswono

Department of Mathematics, The State University of Surabaya
Jl. Ketintang, Surabaya, Jawa Timur 60231 – Indonesia
e-mail : tatagyes@yahoo.com

Abstract. *Recently, learning mathematics doesn't enough just find a solution of routine problems. Students should have skills and competence to solve non routine or open ended problems. Open ended problems can promote mathematical creativity, improve their understanding, and motivate students to develop their ideas. Creativity is an important point in education not only in mathematics because it is needed to solve daily problems. The rapidly changing of technological and a limited of natural resources encourage someone to be creative and no surrender in facing daily life. Therefore, it need a tool to practice and direct students especially in learning mathematics by problem solving of open ended problems.*
This paper will discuss a role of open ended problems to promote mathematical creativity and some example of problems.
Keywords*: problem solving, open-ended problem, creativity*

1. Introduction

The National Curriculum (2006) stated that mathematics is given to all students from elementary school to carry through them among others to empower the creative thinking ability. Creativity is an important point in education not only in mathematics because it is needed to solve daily problems. The rapidly changing of technological and a limited of natural resources encourage someone to be creative to solve reality problems. Therefore, it need a tool to practice and direct students especially in learning mathematics by problem solving of open ended problems. Problem solving approach was a focus in mathematics teaching and learning that covered closed problems with one solution, open-ended problem with more one solution, and problem with various methods for solving (Content Standard of Mathematics Curriculum, 2006).
Traditional problems in mathematics teaching and learning have a common feature that one and only one correct answer. The problems are also well formulated and the correct answer one is unique. That is called "complete" or "closed" problems. This paper will discuss a role of open ended problems to promote mathematical creativity and some example of problems.
The importance of open-ended problems lies in the fact that they break the stereotype that every problem has one correct solution. They also enable each student to work on the same problem according to his or her abilities. However, the primary importance of problems of this kind lies in the fact that they can be used to learn various strategies and thus deepen the students' mathematical knowledge and develop their creative mathematical thinking.

2. Open-Ended Problems

In mathematics teaching and learning, we commonly accepted definition of a "problem" as one where thinking takes place when a person is confronted with a situation or a question that has no immediate solution and that the problem solver accepts the challenge to tackle it. That definition as said by Bell (1981) that a problem is a situation (a question or issue) which needs to be dealt with. Lester (1980) also stated a problem is a situation in which an individual or group is called upon to perform a task for which there is no readily accessible algorithm which determines completely the method of solution. Then, problem solving is the set of actions taken to perform the task.

Bell (1981) suggested four characteristics of a problem. First, a person must become aware of a situation in order for it to be a problem for him or her. Second, she or he must recognize the fact that the situation requires some actions. Third, the person must either need to or want to act upon the situation and must actually take some action. Fourth, the resolution of the situation must not immediately obvious to the person who acts upon it. Another word, a situation or question becomes a problem for someone, if they have a previous or initial knowledge, it challenges them, and algorithm of the method of solution is not immediately obvious. We shall use these characteristics to indicate a situation (question or issue) related mathematics as a mathematics problem.

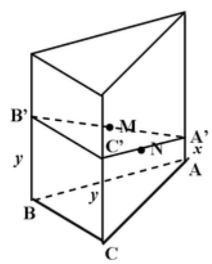
A mathematics problem in teaching and learning procces is applied as a task to direct students developing mathematical skills. They could be clasified as an open or closed problem. Pehkonen (2007) defined tasks are said to be open, if their starting or goal situation is not exactly given. Open is an opposite to closed, in the sense that a task is said to be closed, if the starting situation is exactly given, i.e. the task is well-defined, and if there is one certain result. In open tasks, pupils are given freedom, possibly even in the posing of the question, but at least in the solving of the task. In practice this means that they may end up with different, but equally correct solutions, depending on the additional choices made and the emphases placed during their solution processes. Therefore, open tasks usually have several correct answers.

Ejersbo (2003) define opennes problems in mathematics learning as: (1) the process is open, which mean that you can choose different strategies to reach the goal; (2) the end-products are open, which mean that you have several result to choose among; (3) the ways of formulation problems are open, which mean that you make the question and tasks yourself. Based on it, we could conclude that open ended problem will have multiple possible answers that can be derived by multiple solution methods. The focus is not on the answer to the problem, but on the methods for arriving at an answer.

In open-ended problem solving, students are responsible for making many of the decisions that, in the past, have been the responsibility of teachers and textbooks. To decide which method, or procedure, to undertake to solve an open-ended problem, students will draw on their previous knowledge and experience with related problems. They might construct their own procedure, trying this and that, before arriving at a solution. Then, they will reflect on and explain to others problem-solving experience, tracing their thinking process and reviewing the strategies. This period of reflection deepens his/her understanding of the problem and helps to clarify their thinking about effective solution methods, and how the problem and methods he/she used relate to other problems or areas of mathematics.

The open-ended problems could be related with the context. de Lange (1987) supposed that the context problem serve both as a source for conceptual mathematization and as a field for application of mathematical concepts. For teaching and learning process, we think that a problem, including open-ended problem, has three roles. They enable as a motivation, a source for conceptualize of mathematics concept, and as a field of mathematics application. As a motivation actually we always uses as introduction in teaching and learning process by explain about the importance of subject matter in daily life. As a source for conceptualize of mathematics, we might put a problem as starting point to straight forward students in constructing mathematics concepts. After students understand a mathematical concept, a teacher might extent students' understanding and broad the relationship of mathematical concepts with other discipline by applying it for contextual problems. de Lange (1987) categorize the uses of context in a problem. They are three order context uses. First order context use if the context in this situation is only used to camouflage' the mathematical problem. For example is the frog in the well problem. "A frog is at the bottom of a 10 m well. Each day during the daylight hours the frog climbs up 3 m. At night when it is sleeping it slides back down 2 m. How many days does it take for the frog to get out of the well?". de Lange (1987) has given "the growth of bacteria" problem that is "the growth factor of a bacterium type is equivalent to 6 (per time-unit). At moment 0 there are 4 bacteria. Calculate the point in time when there will be 100 bacteria". Those problems are open-ended problem because students could use multiple strategies to solving them. Second order context use, if a problem is a lot open to the students but there is no conceptual mathematization although that problem related with a context. For example is given starfish growth picture and information: 1 July, 3; 2 July, 7; 4 July, 15; 5 July, 16; 6 July, 18; 7 July, 26, ext. Question: You see one starfish during twelve phases of its growth. Is there interval where growth is exponential? The second and first order context uses are not always distinguishable will be clear. Both are related with a context but they are different in uses. First order is just used commonly to find

mathematical problem and second order is used to construct a mathematical concept. Third order context use, if a problem is used to be a starting point for introducing the mathematical concept. For example is given an actual graph about the growth of aquatic plants (time in weeks). Question is “estimate by means of the graph after how many days there were about 20 m^2 plants?”. This problem is directed to construct a logarithm concept.
Actually, open-ended mathematics problem is not just related with daily context. We could develop only in mathematics field. As example “What is the next three numbers from the pattern 2, 4, 7? Explain your reasons”. We could classify level of problems based on the openness of problem. First level is a trivial open problem, if the problem just differs in representation and it is commonly recognized or taught in mathematics learning. It is just related with one mathematics concept. For example are “Solve the system of linear equation following” or “Give an expression of quadratics equation”. Those problems are open-ended but they are commonly recognized by students because it is taught in teaching and learning process by using substitution, elimination, or both. Second level is a lot open problem, if the problem sometimes related with “realistic” context and need multiple strategies to solve it or it has various solutions. The frog in the well problem could be an example for this level. Third level is actual open-ended problem, if the problem related with realistic context and it had different strategies or multiple right solutions. If its problem is natural mathematics context, it will connected with other mathematics subject/concepts and had multiple strategies and various solution. As an example is the water–flask problem by Becker and Shimada (1997). “Suppose that we have a water flask in the form of a triangular prism that is half full. The flask is titled while one side of the base is fixed on a tabletop. Many quantitave or qeometric relations invoving various parts of the flask are implicit in this situation. Try to discover as many of them as possible and give the reason why such relation hold”.



The midpoints M and N, of A’B’ and A’C’, respectively

The findings example of all students groups, such as: (1) x+2y = k (AA’ + BB’ + CC’ is constant), (2) The total area of the side planes under the water level is constant, (3) the water surface passes through a fixed point, (4) the shape the water surface is always a triangle, (5) the areas of the wo side faces are equal, (6) Each half of the decrease in AA’ is added to BB’ and CC’, respectevely.
Level of open-ended problem determined complexity of problem and the role of problem to asses the instructional goal including to promote creativity in mathematics.

3. Open-Ended Problem and Creativity in Learning Mathematics

The problem originally is used to evaluate students achievement of the higher-order thinking in mathematics education (Becker and Shimada, 1997). The higher-order thinking covered the critical thinking and creative thinking. So, an open-ended problem might be used to indicate students achievement in creative thinking or creativity in mathematics. Researchers at Japan’s National Institute for Educational Research conducted a six-year research study that evaluated higher-order mathematical thinking using open-ended problems (problems with multiple correct answers). In a round-table review of the study, Sugiyama from Tokyo Gakugei University affirmed this approach as a means to allow students to experience the first stages of mathematical creativity (Mann, 2005).
Wilensky (1993) pointed that open ended activities are good activities, and allow for many different approaches and solutions. Open-ended activiteis encourage learners to pose their own problems and make conjectures as to what mathematical relationships obtain.

Klavir and Herkovitz (2007) also expressed that providing a relatively simple evaluation tool for open-ended mathematical assignments can help both teachers and students to evaluate the solutions and can serve as an important means for encouraging creative mathematical thinking.

Pehkonen (1997) explained that the term "open-ended problems" has very near connections with creativity. In others time, he also said that internationally it is accepted that open-ended problems form a useful tool in the development of mathematics teaching in schools, in a way that emphasizes understanding and creativity. One way to provide pupils with such opportunities that stimulate higher-order thinking is to let them carry out investigations, especially open-ended investigations, where pupils pose the problem to be investigated and design their own procedures to answer the question (Pehkonen, 2007). Based on argument above, we could conclude that an open ended task (problem) enable promote student's creative thinking or their creativity in mathematics. Then, what is the meaning of creativity in mathematics classroom?

Creativity in mathematics is different with characteristics creativity in arts or others discipline. Creativity in mathematics should involve characteristics and concepts of mathematics. If someone said that $\frac{1}{2}+\frac{1}{3}=\frac{2}{5}$, he/she is not creative; although it is a result of his/her original thought or it is unusual.

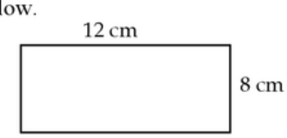
There are many definitions of creativity in the relevant literature. Bergstrom (in Pehkonen, 1997) explained that creativity is performance where the individual is producing something new and unpredictable. According Cropley (in Haylock, 1997) that creativity refers to a special kind of thinking or mental functioning, often called divergent thinking. In terms of teaching children in schools, Cropley leans that creativity is the capacity to get ideas, especially original, inventive, and novel ideas. Haylock suggested that Cropley approach of creativity is most prevalent in discussion about creativity in school mathematics. The focus is on identifying the kinds of thinking in mathematical tasks that qualify for description "creative". To know who are creative, it uses divergent production tasks. In these tasks a student is given open-ended situation to which many possible responses may be made, so we can show divergent thinking in their response.

Some researchers have different approach to identify students' creativity. Klavir & Hertzovit (2007) analyze the indices of creativity in the spirit of Guilford and Torrance according to four components: Fluency – A person's ability to extract a large number of solutions that meet the limitations of the assignment. Flexibility – A person's ability to shift from one way of thinking to another and extract solutions that refer to different categories. Elaboration – A person's ability to elaborate the given idea, add details to it, develop it by means of a combination of additional ideas, and/or refine it. Originality – A person's ability to approach the given problem in a new and unique way and extract unexpected and unconventional solutions. In the mathematical context the fluency dimension can illuminate students' amount of active and available knowledge with reference to a given mathematical task. The flexibility dimension can examine, as it does in the creativity test, the extent to which learners can shift from one state of thinking to another when solving open-ended mathematical problems. This dimension reflects the learners' flexibility in utilizing a different mathematical principle every time, the numbers different mathematical properties, and so on. The third dimension, elaboration, can indicate complexity of mathematical thinking: more complex solutions reflect a more complex ability to integrate different pieces of mathematical knowledge. And finally, the fourth dimension, originality, examines creativity from the perspective of identifying what is unique in a particular student's solution in comparison to the common and prevailing solutions among his or her peers. This dimension can serve as a criterion for student's original mathematical thinking; their ability to attack an open-ended mathematical problem from new and unexpected angles.

Silver (1997) adapted the Torrance Tests of Creative Thinking to assess the creative thinking of students. He just use the notion of fluency, flexibility, and novelty to applied in domain of mathematics. Problem posing and problem posing tasks have been used to identify creative individuals. In problem solving, students indicate fluency, if they could explore open-ended problems with many interpretations, solutions methods, or answer. Flexibility, if students solve (or expresss or justify) in one way, then in other ways. Students also discuss many solution methods. Novelty, if students examine many solution methods or answer (expression or justification), then generate another that is different. In problem posing, students indicate fluency if they generate many problems to be solved. Students also share their posed problems. Flexibility, if students pose problems that are solved in different ways or students use "what-if-not" approach to pose problems. Novelty, if students examine several posed problems, then pose a problem that is different. That indicators are appropriate to assess students' creative thinking in mathematics

because they are suitable with mathematics characteristics and used by many mathematics researcher as identifying student creative performances. Elaboration indicator is not applied because it is covered in others indicator. Another reason because its definition of creativity emphasize to a product of creativity.
This approach is applied by Siswono (2007) to develop the level of students creative thinking in problem solving and problem posing. There are five hierarchical levels of students' creative thinking in mathematics with different characteristic for each level. Characteristics based on the indicator of creative thinking (fluency, flexibility, and novelty). Fluency in problem solving refers to a student's ability to obtain various solutions of a problem. Fluency in problem posing refers to student's ability to pose various problems with a correct solution. Flexibility in problem solving refers to a student's ability to solve a problem using many different methods or ways. Flexibility in problem posing also refers to a student's ability to pose or construct problem with divergent solutions. Novelty in problem solving refers to the student's ability to solve a problem with many different solutions and correct answers, especially to find an original solution which is not common for that student's grades or their knowledge level. Some solutions are different if they have a different pattern or are not usual for student grades, such as students can construct a combination of other shapes. Novelty in problem posing refers to a student's ability to pose or construct the different problems with initial problem. The differences might be in contexts or concepts which are used in a problem.
The following is an example task to encourage student's creative thinking.

Area of Rectangle
It is given a rectangle in below.



a. Construct polygons which their areas are equal with a rectangle above!
b. Draw at least two another polygons which their area are equal with that rectangle!
c. Look at one of your polygons. Show different strategies or methods to find your solution.
d. Write at least two different problems which correlated with given rectangle and give your solutions.
e. Based on your problems, is there a problem with multiple strategies to find the solution? If you have made it, show another different strategy to solve it. If there is no one a problem, construct another problem which it has multiple strategies or methods.

First question ask student to solve a problem about constructing polygons. If they can overcome the first question, it means they understand the problem solving procedures and show 'fluency'. Second question (item b) is an indicator to identify 'novelty' in problem solving and third question (item c) as indicator of flexibility in solving problem. Fourth and fifth questions are indicators of creativity in mathematics problem posing. Fourth question show indicator of fluency and novelty. Then, fifth question as indicator of flexibility in creating problems.
If that format of open-ended problem is always implemented in mathematics teaching, we believe that will give effect for increasing higher order thinking and understanding mathematical concepts. Based on my research, flexibility in posing and solving problems is difficult aspect to be increased. It might need an enough time to perform flexibility.

4. Summary

In teaching and learning mathematics, creativity still is not emplemented or focused by teacher. Teacher do not keep attention to effort improving student's creative thinking. That is happened because some reasons; first they do not realize that the Indonesian Curriculum (2006) for mathematics empowered the creative thinking ability. They just look at the content standard or subject matters. They think that all subject matter is compulsory finished and taught at recently semester. Therefore, their teaching process only emphasizes recognition or reinforcement. Second, they do not know about creativity in mathematics. Creative commonly related with art or poetry. They think commonly that mathematics and creativity is different perspective. Mathematics ask precisely and exactly in all aspect, but creativity is not logic and just intuition. Third, they do not know how to teach or manage their classroom so that encourage creative atmosphere. Fourth, they do not have a vision how their students future after learning at formal school and going back to real life. Because the national system of assessment just focused at routine problem and used technique paper and pencil test by multiple choice, then they just compassed that in teaching and learning process.

To overcome some problems, we must promote the importance of creativity in mathematics teaching and learning. We could apply an open-ended problem solving and prepare examples how to teach it in mathematics classroom. An open-ended problem request students to solve with multiple strategies or method, and the ended solution is various.

The open-ended problem should be taught step by step for pre-stage using first level problem. Student is given an opportunity to adapt this problem and started by mathematics simple problem. If it is enable, we could use a "camouflage" or realistic context. An open-ended problem could be ranked as level 1, level 2, and level 3. Level one is a trivial problem, and level two contain a lot open problem. Level two has multiple strategies or multiple solutions without both, but level three contain both features.

To promote mathematical creativity, students should be recognized by divergent (open-ended) problems with multiple strategies and various solutions. Teacher also is asked over to be creative in developing lesson plan and must learn to solve challenging mathematics problems.

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